4 Complexity Theory (AD)

Consider the following two decision problems:

- **Reach** – the problem of deciding, given a *directed* graph G and two vertices a and b in G, whether there is a path in G from a to b.

- **UReach** – the problem of deciding, given an *undirected* graph G and two vertices a and b in G, whether there is a path in G from a to b.

It is known that **Reach** is NL-complete (under logarithmic-space reductions) and that **UReach** is in the complexity class L.

(*a*) Based on the above information, for each of the following statements, state whether it is true, false, or unknown. In each case, give justification for your answer and in the case where the truth of the statement is unknown, state any implications that might follow from it being true or false.

(*i*) **Reach** $\leq_L$ **UReach**, i.e. **Reach** is reducible in logarithmic-space to **UReach**.

(*ii*) **UReach** $\leq_L$ **Reach**.

(*iii*) **UReach** is in P.

(*iv*) If **Reach** is in L, then P=NP.

[3 marks each]

(*b*) Let us say that a nondeterministic Turing machine M is *symmetric* if for any two configurations c_1 and c_2 of M, if $c_1 \to_M c_2$, then $c_2 \to_M c_1$. We write SL for the class of all languages that are accepted by a symmetric Turing machine using $O(\log n)$ work space on inputs of length n.

By considering the configuration graph of a machine and using the fact that **UReach** is in L, explain why it follows that SL⊆L. [8 marks]