

8 Discrete Mathematics (MPF)

(a) Let  $R \subseteq X \times Y$  and  $P \subseteq Y$  for sets  $X$  and  $Y$ .

Prove that

$$\forall y \in Y. ( [(\exists x \in X. x R y) \Rightarrow y \in P] \iff [\forall x \in X. (x R y \Rightarrow y \in P)] )$$

[6 marks]

(b) Define the notions of

(i) injective function between two sets [1 mark]

(ii) surjective function between two sets [1 mark]

(c) Let  $\mathbb{N}_+ = \{n \in \mathbb{N} \mid n > 0\}$  and define the function  $e : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}_+$  by

$$e(m, n) = 2^m(2n + 1)$$

Without using the Fundamental Theorem of Arithmetic, prove that  $e$  is

(i) injective [4 marks]

(ii) surjective [8 marks]

You may use any other standard results provided that you state them clearly.