

7 Discrete Mathematics (MPF)

(a) Find all solutions in  $\mathbb{Z}_{187}$  of the following congruence

$$x^2 + 5x + 6 \equiv 0 \pmod{187}$$

Justify your answer.

[6 marks]

(b) For  $\ell \in \mathbb{N}$ , let  $[\ell] = \{i \in \mathbb{N} \mid i < \ell\}$ .

(i) Prove that, for all  $\ell, m \in \mathbb{N}$ ,  $[m] \times [\ell] \cong [m \cdot \ell]$

[3 marks]

(ii) Prove that, for all  $\ell, m \in \mathbb{N}$ ,  $[m] \uplus [\ell] \cong [m + \ell]$

[3 marks]

(iii) For  $m, n \in \mathbb{N}$ , define  $\oplus$  by

$$[m] \oplus [0] = [m] \quad \text{and} \quad [m] \oplus [n + 1] = ([m] \oplus [n]) \uplus [1]$$

Prove that, for all  $\ell, m \in \mathbb{N}$ ,

$$[m] \oplus [\ell] \cong [\ell] \oplus [m]$$

[8 marks]

You may use any standard results provided that you state them clearly.