10 Discrete Mathematics (IML)

(*a*) Let $\Sigma = \{a, b\}$ and let $\#_x(w)$ be the number of occurrences of the symbol $x \in \Sigma$ in the string w. For each of the following determine, with justification, whether or not the language is regular.

(*i*) $L_1 = \{w \in \Sigma^* \mid \#_a(w) = \#_b(w)\}$ [3 marks]

(*ii*) $L_2 = \{w \in \Sigma^* \mid w$ has an equal number of occurrences of the substrings ab and $ba\}$. [3 marks]

(*iii*) L_3 inductively defined by the following axiom and rule:

$$\frac{}{\epsilon} \qquad \frac{u}{aub} \qquad \text{for all } u \in \Sigma^*$$ [3 marks]

(*iv*) L_4 inductively defined by the following axiom and rules:

$$\frac{}{\epsilon} \qquad \frac{u}{aub} \qquad \frac{u}{au} \qquad \frac{u}{ub} \qquad \text{for all } u \in \Sigma^*$$ [3 marks]

(*v*) $L_5 = \{w \in \Sigma^* \mid (\#_a(w) = 3i) \wedge (\#_b(w) = 7j) \text{ for some } i, j \in \mathbb{N}\}$ [3 marks]

(*b*) Consider the set R of all regular expressions over the alphabet $\{a, b\}$.

(*i*) Give an alphabet sufficient to express any element of R. [2 marks]

(*ii*) State, giving reasons, whether R is a regular language. [3 marks]