6 Denotational Semantics (MPF)

Consider the concocted language PCF∗ obtained from the language PCF by extending it with:

- Types
  \[ \tau ::= \cdots \mid \tau \ast \tau \]

- Expressions
  \[ M ::= \cdots \mid \text{pair}(M, M) \mid \text{left}(M) \mid \text{right}(M) \]

- Typing rules
  \[
  \begin{aligned}
  & \Gamma \vdash M_1 : \tau_1 \quad \Gamma \vdash M_2 : \tau_2 \\
  \implies & \quad \Gamma \vdash \text{pair}(M_1, M_2) : \tau_1 \ast \tau_2 \\
  & \Gamma \vdash M : \tau_1 \ast \tau_2 \\
  \implies & \quad \Gamma \vdash \text{left}(M) : \tau_1 \\
  & \Gamma \vdash \text{right}(M) : \tau_2
  \end{aligned}
  \]

- Values
  \[ V ::= \cdots \mid \text{pair}(V, V) \]

- Operational semantics
  \[
  \begin{aligned}
  & M_1 \Downarrow_{\tau_1} V_1 \quad M_2 \Downarrow_{\tau_2} V_2 \\
  \implies & \quad \text{pair}(M_1, M_2) \Downarrow_{\tau_1 \ast \tau_2} \text{pair}(V_1, V_2) \\
  & M \Downarrow_{\tau_1 \ast \tau_2} \text{pair}(V_1, V_2) \\
  \implies & \quad \text{left}(M) \Downarrow_{\tau_1} V_1 \\
  & M \Downarrow_{\tau_1 \ast \tau_2} \text{pair}(V_1, V_2) \\
  \implies & \quad \text{right}(M) \Downarrow_{\tau_2} V_2
  \end{aligned}
  \]

(a) Give a denotational semantics for the above extension of PCF. \[3 \text{ marks}\]

(b) Show that the denotation of types are domains and that the denotation of terms are continuous functions. You may use any standard results provided that you state them clearly. \[5 \text{ marks}\]

(c) State the soundness property for a denotational semantics of PCF∗. \[2 \text{ marks}\]

(d) Show that your denotational semantics of PCF∗ is sound. You may use any standard results provided that you state them clearly. \[4 \text{ marks}\]

(e) State the adequacy property for a denotational semantics of PCF∗. \[2 \text{ marks}\]

(f) Establish whether or not your denotational semantics of PCF∗ is adequate. You may use any standard results provided that you state them clearly. \[4 \text{ marks}\]