6 Denotational Semantics (MPF)

Consider the concocted language PCF_* obtained from the language PCF by extending it with:

- Types
  $\tau ::= \cdots \mid \tau * \tau$
- Expressions
  $M ::= \cdots \mid \mathbf{pair}(M, M) \mid \mathbf{left}(M) \mid \mathbf{right}(M)$
- Typing rules

$$\frac{\Gamma \vdash M_1 : \tau_1 \quad \Gamma \vdash M_2 : \tau_2}{\Gamma \vdash \mathbf{pair}(M_1, M_2) : \tau_1 * \tau_2} \qquad \frac{\Gamma \vdash M : \tau_1 * \tau_2}{\Gamma \vdash \mathbf{left}(M) : \tau_1} \qquad \frac{\Gamma \vdash M : \tau_1 * \tau_2}{\Gamma \vdash \mathbf{right}(M) : \tau_2}$$

- Values
  $V ::= \cdots \mid \mathbf{pair}(V, V)$
- Operational semantics

$$\frac{M_1 \Downarrow_{\tau_1} V_1 \qquad M_2 \Downarrow_{\tau_2} V_2}{\mathbf{pair}(M_1, M_2) \Downarrow_{\tau_1 * \tau_2} \mathbf{pair}(V_1, V_2)}$$

$$\frac{M \Downarrow_{\tau_1 * \tau_2} \mathbf{pair}(V_1, V_2)}{\mathbf{left}(M) \Downarrow_{\tau_1} V_1} \qquad \frac{M \Downarrow_{\tau_1 * \tau_2} \mathbf{pair}(V_1, V_2)}{\mathbf{right}(M) \Downarrow_{\tau_2} V_2}$$

(*a*) Give a denotational semantics for the above extension of PCF. [3 marks]

(*b*) Show that the denotation of types are domains and that the denotation of terms are continuous functions. You may use any standard results provided that you state them clearly. [5 marks]

(*c*) State the soundness property for a denotational semantics of PCF_*. [2 marks]

(*d*) Show that your denotational semantics of PCF_* is sound. You may use any standard results provided that you state them clearly. [4 marks]

(*e*) State the adequacy property for a denotational semantics of PCF_*. [2 marks]

(*f*) Establish whether or not your denotational semantics of PCF_* is adequate. You may use any standard results provided that you state them clearly. [4 marks]