1 Advanced Algorithms (TMS)

(a) What are the three possible cases for the solution of a linear program? For each of them, give an example of a linear program in standard form exhibiting this case. [6 marks]

(b) What is the set of optimal solutions for the following linear program?

\[
\text{Minimize } -x_1 - x_2 \\
-x_2 \geq -3 \\
2x_1 + x_2 \leq 8 \\
x_1, x_2 \geq 0
\]

[6 marks]

(c) For a given linear program \(\text{LP}_1\)

\[
\text{Maximize } \sum_{j=1}^{n} c_j x_j \\
\sum_{j=1}^{n} a_{ij} x_j \leq b_i \quad (1 \leq i \leq m) \\
x_j \geq 0 \quad (1 \leq j \leq n),
\]

consider a new linear program \(\text{LP}_2\):

\[
\text{Minimize } \sum_{i=1}^{m} b_i y_i \\
\sum_{i=1}^{m} a_{ij} y_i \geq c_j \quad (1 \leq j \leq n) \\
y_i \geq 0 \quad (1 \leq i \leq m).
\]

(i) Prove that if \(x\) is a feasible solution for \(\text{LP}_1\) and \(y\) is a feasible solution for \(\text{LP}_2\), then \(c^T x \leq b^T y\). [6 marks]

(ii) Using your answer in Part (c)(i), what can we conclude about \(\text{LP}_2\) if we know that \(\text{LP}_1\) is unbounded? [2 marks]