5 Numerical Methods (DJG)

(*a*) Consider implementing the natural logarithm function $\ln(t)$ for floating-point numbers using the McLaurin series:

$$\begin{aligned}\ln(1+x) &= \sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n} \\ &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots\end{aligned}$$

(*i*) List all special behaviours the natural logarithm function should have in different parts of its range and when t takes the special values NaN and $\pm\infty$. [2 marks]

(*ii*) The function must accept a broad range of numerical values but the series only converges when the absolute value of x is less than one, $|x| < 1$. Describe a range-reduction procedure that pre-processes the argument and post-processes the result so that the series always acts on small values of x. [6 marks]

(*iii*) State the two precision requirements normally expected for mathematical libraries. Considering the worst-case value(s) of x after range reduction, approximately how many terms are needed to meet one of these requirements for a single-precision implementation? Do you expect the other requirement to be met? [6 marks]

(*b*) The Trapezoidal Rule for numerical definite integration returns the area of the trapezium-shaped strips formed by each pair of adjacent points. The area under each such strip is:

$$\int_a^b f(x)\,dx \approx \tfrac{b-a}{2}\,[f(a) + f(b)]$$

(*i*) A program computes the area between two points A and B using N strips of width h. What should be taken into account when choosing h? Suggest a good value for h. [3 marks]

(*ii*) Assuming the best choice for h, what characteristics of $f()$ will affect the accuracy achieved? [3 marks]