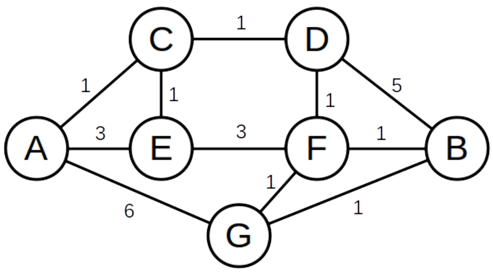
10 Algorithms (RKH/DJW)

(*a*) Let `dijkstra_path`(g, a, b) be an implementation of Dijkstra's shortest path algorithm that returns the shortest path from node a to node b in a graph g. Prove that the implementation can safely terminate when it first encounters node b. [5 marks]

(*b*) Consider all paths in a graph from a to b, ordered from shortest to longest. Assuming $p =$ `dijkstra_path`(g, a, b) is the first path in this collection, an algorithm to find the second path considers deviations from the vertices of p. An algorithm to do this is given below.

```
function second_path(Graph g, Vertex a, Vertex b):
   p = dijkstra_path(g,a,b)
   best_so_far = []
   for i = 1 to len(p)-1:
      t = p[:i]     # First i elements of p
      c = g.get_edge_weight(p[i], p[i+1])
      g.set_edge_weight(p[i], p[i+1], infinity)
      t.append(dijkstra_path(g,p[i],b))
      if (len(best_so_far) == 0 or
          cost(t) < cost(best_so_far)):
        best_so_far = t
      g.set_edge_weight(p[i], p[i+1], c)
   return best_so_far
```

(*i*) Show the steps of this algorithm on the following graph, from A to B.



[5 marks]

(*ii*) What is the asymptotic complexity of this algorithm in terms of the number of edges, E, and the number of vertices, V? Assume the implementation of Dijkstra's algorithm uses a priority queue based on a Fibonacci heap. [4 marks]

(*iii*) Show how to adapt this algorithm to find the top-k shortest paths in the collection. State the complexity of the adapted algorithm. [6 marks]