3 Computer Systems Modelling (IML)

Consider the simulation of a simple server with an unspecified arrival process and and unspecified service distribution. Observations of when customers arrive into the queue, and when they complete service, are readily available. Service is strictly first-come-first-served.

(*a*) As an initial step it is assumed that the arrival process is Poisson with arrival rate λ.

(*i*) Describe the inverse transform method for generating continuous random variables with specified distributions, assuming a source of random variables from $U(0, 1)$, that is, uniformly distributed on the interval $[0 : 1]$. [5 marks]

(*ii*) Apply this method to generate random variables representing the time intervals between arrivals. [3 marks]

(*iii*) A student generates arrival events by dividing time into small intervals Δt such that $\lambda\Delta t \ll 1$ and takes a sample u from $U[0, 1)$. If $u < \lambda\Delta t$ then an arrival is generated, otherwise not. Is this a good way of generating Poisson arrivals with rate λ? Explain your answer. [2 marks]

(*b*) In order to investigate the arrival process, observations of arrivals into the system are gathered and stored as a sequence $Y_1, Y_2, \ldots, Y_n$ of time intervals between arrivals.

(*i*) Given an assumption that the arrival process is Poisson, how could λ be estimated? [2 marks]

(*ii*) How might one use the Kolmogorov-Smirnov test to test the hypothesis that the arrival process is Poisson? [8 marks]