16 Types (AMP)

(*a*) Existential types can be encoded in the Polymorphic Lambda Calculus (PLC) by defining $\exists\,\alpha\,(\tau)$ to be $\forall\beta\,((\forall\alpha\,(\tau\to\beta))\to\beta)$, where $\beta \neq \alpha$ and β does not occur free in the PLC type τ. Give the following definitions, justifying the typings in each case:

(*i*) A closed PLC term `pack` of type $\forall\alpha\,(\tau \to \exists\,\alpha\,(\tau))$. [5 marks]

(*ii*) A PLC term $\texttt{unpack}(M, x, M', \tau')$ satisfying $\Gamma \vdash \texttt{unpack}(M, x, M', \tau') : \tau'$ whenever $\Gamma \vdash M : \exists\,\alpha\,(\tau)$ and $\Gamma, x : \tau \vdash M' : \tau'$ hold, where x is not in the domain of Γ and α does not occur free in Γ or τ'. [5 marks]

(*b*) If $\Gamma \vdash M : \exists\,\alpha\,(\tau)$, $\Gamma, x : \tau \vdash M' : \tau'$ and $\Gamma \vdash N : \tau[\tau'/\alpha]$ hold, where x is not in the domain of Γ and α does not occur free in Γ or τ', to what term does $\texttt{unpack}((\texttt{pack}\,\tau'\,N), x, M', \tau')$ beta-reduce? [2 marks]

(*c*) For each PLC type τ, let $\neg\tau$ be the type $\tau \to \forall\alpha\,(\alpha)$. Give, with justification, closed PLC terms of the following types

(*i*) $\forall\alpha\,(\neg\tau) \to \neg\exists\,\alpha\,(\tau)$ [4 marks]

(*ii*) $\exists\,\alpha\,(\neg\tau) \to \neg\forall\alpha\,(\tau)$ [4 marks]