15 Topics in Concurrency (JMH)

Let the pure CCS processes P_1, P_2, Q_1 and Q_2 be as follows.

$$P_1 \stackrel{\text{def}}{=} a.P_1 + \bar{b}.Q_1 \qquad\qquad Q_1 \stackrel{\text{def}}{=} b.P_1$$

$$P_2 \stackrel{\text{def}}{=} b.Q_2 \qquad\qquad Q_2 \stackrel{\text{def}}{=} c.Q_2 + \bar{b}.P_2$$

The transition system from $(P_1 \parallel P_2) \setminus \{b\}$ is as follows.

$$\begin{tikzcd}
(P_1 \parallel P_2) \setminus \{b\} \arrow[loop above, "a"] \arrow[r, bend left, "\tau"] & (Q_1 \parallel Q_2) \setminus \{b\} \arrow[loop above, "c"] \arrow[l, bend left, "\tau"]
\end{tikzcd}$$

(a) Give full derivations for the two transitions that start from $(P_1 \parallel P_2) \setminus \{b\}$. [5 marks]

(b) The full modal-μ calculus has the syntax

$$A ::= T \mid S \mid \neg A \mid A_1 \wedge A_2 \mid A_1 \vee A_2 \mid \langle a \rangle A \mid [a]A \mid \nu X.A \mid \mu X.A \mid X,$$

where S is an arbitrary set of states. Give a semantics to closed formulas *without* using the abbreviations $\mu X.A \equiv \neg \nu X.\neg A[\neg X/X]$ and $[a]A \equiv \neg\langle a \rangle \neg A$. What condition must be placed on the occurrence of variables and why? [5 marks]

(c) Prove that the operation

$$X \mapsto [a]X$$

is $\bigcap$-continuous. [5 marks]

(d) Give a modal-μ formula that is satisfied by a process if, and only if, it is bisimilar to the process $(P_1 \parallel P_2) \setminus \{b\}$. You may assume that the process is only capable of actions labelled a, c and τ. [5 marks]