Recall that basic Petri nets can be considered to be general Petri nets where all multiplicities (including the capacity of conditions) are less than or equal to 1, so the initial marking and pre- and post-conditions form sets.

(a) Give the token game for basic Petri nets, specifying when $\mathcal{M} \xrightarrow{e} \mathcal{M}'$. [3 marks]

(b) Draw basic Petri nets to illustrate the concepts of forwards conflict, backwards conflict and independence (also called concurrency). [4 marks]

(c) Draw a basic Petri net for which the token game gives the following transition system:

\[
\begin{align*}
\mathcal{M}_0 & \quad \xrightarrow{a} \quad \mathcal{M}_1 \\
& \quad \xrightarrow{b} \quad \mathcal{M}_2 \\
\mathcal{M}_1 & \quad \xrightarrow{c} \quad \mathcal{M}_3 \\
\mathcal{M}_2 & \quad \xrightarrow{d} \quad \mathcal{M}_3 \\
\end{align*}
\]

The initial marking should be $\mathcal{M}_0$ and your answer should have four events, $a, b, c$ and $d$. [5 marks]

(d) Show how conditions can be added to an arbitrary basic net to create a safe net with the same behaviour.

Your answer should describe what a safe net is, give the construction generally and prove briefly why the net obtained is safe.

*Hint:* systematically add conditions that represent the vacancy of conditions in the original net. [8 marks]