3 Machine Learning and Bayesian Inference (SBH)

(a) For random variables (RVs) $A_1$, $A_2$ and $B$, define what it means for $A_1$ to be *conditionally independent of* $A_2$ *given* $B$, written $A_1 \perp A_2 | B$. [1 mark]

(b) Given mutually disjoint sets $X_1$, $X_2$ and $Y$ of random variables from some Bayesian network, define what it means to say that a path from $x_1 \in X_1$ to $x_2 \in X_2$ is *blocked* by $Y$. [5 marks]

(c) Given mutually disjoint sets $X_1$, $X_2$ and $Y$ of random variables from some Bayesian network, define what it means for $X_1$ and $X_2$ to be *d-separated* by $Y$. What does this tell you about the probability distribution represented by the Bayesian network? [3 marks]

(d) Consider the following Bayesian network.

In each of the following cases, establish whether or not $X_1 \perp X_2 | Y$:

(i) $X_1 = \{ A, E \}$, $X_2 = \{ G \}$ and $Y = \{ F \}$. [2 marks]

(ii) $X_1 = \{ A \}$, $X_2 = \{ D \}$ and $Y = \{ G \}$. [3 marks]

In each case explain your answer.

(e) Define how Bayesian networks and Markov random fields are used to represent probability distributions, and briefly describe the trade-offs involved in choosing one versus the other. [6 marks]