1 Advanced Algorithms (TMS)

(a) Into what three cases can a linear program in standard form be classified? [3 marks]

(b) Consider the (unweighted) vertex cover problem for the graph $G = (V, E)$ with $V = \{1, 2, 3\}$ and $E = \{\{1, 2\}, \{2, 3\}, \{1, 3\}\}$.

(i) Write down the linear program relaxation for the vertex cover problem and solve the linear program. [6 marks]

(ii) Based on the solution of the linear program in (b)(i), derive an integer solution using the rounding approach described in the lecture. [2 marks]

(c) Consider the following randomised algorithm for the unweighted vertex cover problem:

Initialize $S$ to be the empty set
For all edges $e = (u, v)$ do
  If neither $u$ nor $v$ belongs to $S$
    Randomly choose $u$ or $v$ with probability $1/2$
    and add the vertex to $S$
  End If
End For
Return $S$

Derive an upper bound, as tight as possible, on the approximation ratio of the algorithm.

*Hint:* Try to find an invariant that bounds from below the size of the intersection of the current solution $S = S(i)$ with the optimum solution, where $S(i)$ denotes the set $S$ after the $i$-th iteration of the FOR loop. [9 marks]