8 Mathematical Methods for Computer Science (RJG)

(*a*) (*i*) State the central limit theorem. [2 marks]

(*ii*) Consider a binomially distributed random variable T with parameters Bin(n,p) where n is a positive integer and $0 < p < 1$. Using the central limit theorem derive an approximation to the probability $\mathbb{P}(T > d)$ where $d \in (0, n)$ and where n is sufficiently large. [4 marks]

(*b*) Let $(X_n)_{n\geq 1}$ be a Markov chain on the states $\{0, 1, 2\}$ with transition matrix

$$P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1-\alpha & \alpha \\ 1-\alpha & \alpha & 0 \end{pmatrix}$$

where $0 < \alpha < 1$.

(*i*) Draw the state space diagram for the Markov chain X_n. [2 marks]

(*ii*) Explain why X_n is an irreducible, recurrent and aperiodic Markov chain. [6 marks]

(*iii*) Define an equilibrium distribution $\pi = (\pi_0, \pi_1, \pi_2)$ for the Markov chain X_n and determine π. [6 marks]