6 Logic and Proof (LCP)

(a) Draw ordered binary decision diagrams (BDDs) for each of the following formulas, thereby identifying which of them are logically equivalent.

\[ P \rightarrow (Q \rightarrow R) \]
\[ P \rightarrow (R \rightarrow Q) \]
\[ (\neg Q \lor R) \lor \neg P \]

[8 marks]

(b) A mysterious propositional connective, \( \odot \), has the following right-side sequent calculus rule, \((\odot r)\):

\[ \Gamma, A, B \Rightarrow \Delta \]
\[ \Gamma \Rightarrow \Delta, A \odot B \]

Present the corresponding left-side sequent calculus rule, \((\odot l)\), along with the truth table for \( \odot \). [6 marks]

(c) For the following formula, either exhibit a formal proof (using the sequent calculus, augmented with the \((\odot r)\) rule above) or exhibit a falsifying interpretation:

\[ \Rightarrow \exists x (P(x) \odot Q(x)), (\forall x P(x)) \land (\forall x Q(x)) \]

[6 marks]