5 Logic and Proof (LCP)

(a) Exhibit an interpretation in S4 modal logic that simultaneously satisfies the formulas $P \land Q$, $\Box(P \lor Q)$, $\Diamond \neg P$, $\Diamond \neg Q$ at a particular world, $w$. [5 marks]

(b) For each of the following sets of clauses, either exhibit a model or show that none exists. Below, $a$ and $b$ are constants, while $x$, $y$ and $z$ are variables. Briefly justify your answers.

(i) 
\[
\{ \neg R(x, y), R(f(x), f(y)) \} \\
\{ R(a, b) \} \quad \{ \neg R(x, x) \} \\
\{ \neg R(y, x), R(y, z), \neg R(x, z) \}
\]

[7 marks]

(ii) 
\[
\{ \neg Q(x, y), \neg Q(y, x), R(x) \} \\
\{ \neg P(a, y), Q(y, y) \} \\
\{ \neg Q(x, y), P(b, x) \} \\
\{ P(z, b), P(x, y) \} \\
\{ \neg R(b), \neg R(y) \}
\]

[8 marks]