4 Computation Theory (AMP)

(a) Explain what it means for a partial function \( h \) to be defined by *primitive recursion* from partial functions \( f \) and \( g \). Why is \( h \) a totally defined function if \( f \) and \( g \) are? [5 marks]

(b) (i) Define the class of *primitive recursive* functions. [5 marks]

(ii) For each \( n \in \mathbb{N} \), show that the constant function \( \mathbb{N} \to \mathbb{N} \) with value \( n \) is primitive recursive. [2 marks]

(iii) Explain why it is the case that not every function \( \mathbb{N} \to \mathbb{N} \) is primitive recursive, carefully stating any general results you use. [3 marks]

(c) Given \( e \in \mathbb{N}^2 \to \mathbb{N} \) and \( n \in \mathbb{N} \), let \( e_n \in \mathbb{N} \to \mathbb{N} \) be the function given by \( e_n(x) = e(n, x) \). Suppose that \( e \) is primitive recursive.

(i) Show that each \( e_n \) is primitive recursive. [1 mark]

(ii) Using a suitable diagonalisation argument, or otherwise, prove that it cannot be the case that for all primitive recursive functions \( f \in \mathbb{N} \to \mathbb{N} \) there exists \( n \in \mathbb{N} \) with \( e_n \) equal to \( f \). [4 marks]