4 Computation Theory (AMP)

(*a*) Explain what it means for a partial function h to be defined by *primitive recursion* from partial functions f and g. Why is h a totally defined function if f and g are? [5 marks]

(*b*) (*i*) Define the class of *primitive recursive* functions. [5 marks]

(*ii*) For each $n \in \mathbb{N}$, show that the constant function $\mathbb{N} \to \mathbb{N}$ with value n is primitive recursive. [2 marks]

(*iii*) Explain why it is the case that not every function $\mathbb{N} \to \mathbb{N}$ is primitive recursive, carefully stating any general results you use. [3 marks]

(*c*) Given $e \in \mathbb{N}^2 \to \mathbb{N}$ and $n \in \mathbb{N}$, let $e_n \in \mathbb{N} \to \mathbb{N}$ be the function given by $e_n(x) = e(n, x)$. Suppose that e is primitive recursive.

(*i*) Show that each e_n is primitive recursive. [1 mark]

(*ii*) Using a suitable diagonalisation argument, or otherwise, prove that it cannot be the case that for all primitive recursive functions $f \in \mathbb{N} \to \mathbb{N}$ there exists $n \in \mathbb{N}$ with e_n equal to f. [4 marks]