

2 Complexity Theory (AD)

Define DOUBLE-SAT to be the set of Boolean formulas ϕ in CNF such that ϕ is satisfied by *at least two* distinct assignments of truth values to its variables.

Your task is to prove that DOUBLE-SAT is NP-complete. In order to do this, you need to do the following.

- (a) Give a definition of *polynomial-time reductions*. [2 marks]
- (b) Give a definition of the complexity class NP. [2 marks]
- (c) Using parts (a) and (b), give a definition of NP-hardness. [2 marks]
- (d) Using the above, give a definition of NP-completeness. [2 marks]
- (e) Give a proof that polynomial-time reductions are closed under composition. [3 marks]
- (f) Prove that DOUBLE-SAT satisfies all parts of the definition in (d). In your proof, you should make clear where each of the above parts is used. In addition, you may use any standard results about the complexity of Boolean satisfiability, as long as they are clearly stated. [9 marks]