1 Complexity Theory (AD)

Let $S \subseteq \mathbb{N}$ be a set of numbers. We write $\text{bin}S$ for the set of binary strings (i.e. strings in $\{0, 1\}^*$) $x$ such that $x$ is a binary representation of a number in $S$. We also write $\text{un}S$ for the set $\{a^k \mid k \in S\}$ where the notation $a^k$ means the string consisting of $k$ repetitions of the letter $a$.

(a) Suppose $\text{bin}S \in \text{TIME}(2^{cn})$ for some constant $c$. Prove that $\text{un}S \in \text{P}$. [4 marks]

(b) Give definitions of the complexity classes $L$ and $NL$. [4 marks]

(c) Prove that:

(i) $\text{bin}S \in \text{SPACE}(n)$ if, and only if, $\text{un}S \in L$; and

(ii) $\text{bin}S \in \text{NSPACE}(n)$ if, and only if, $\text{un}S \in NL$. [6 marks]

(d) Recall that $\text{Reach}$ is the problem of reachability in directed graphs. Using part (c) or otherwise, show that, if $\text{Reach}$ were in $L$, it would follow that $\text{SPACE}(n) = \text{NSPACE}(n)$. State carefully any standard results that you use. [6 marks]