## 6 Numerical Methods (DJG)

A seismic probe bores itself into the seabed, going as deep as it can before running out of fuel. This takes about five minutes. It rotates its spiral drill head at rate $R(t)$ that follows a pre-programmed profile. Its downwards velocity $\dot{y}$ is proportional to $R$ and to the square-root of the probe's weight $w$, and is given by

$$\dot{y} = 3.6 \times R(t) \times \sqrt{w}.$$

The weight of the probe is the weight of its fuel, which starts at 1500 plus its own intrinsic weight, which is fixed at 35. The fuel weight decreases at a rate of 1.2 $R(t)$.

(*a*) Give the state vector for a forwards FDTD simulation of this system using Euler's Method. [1 mark]

(*b*) Assuming a function is provided that returns $R(t)$, give a program that uses Euler's Method (a straightforward, forward finite-difference simulation) to determine the depth achieved. [4 marks]

(*c*) Describe the errors you might expect if you chose a time step that was inordinately small or inordinately large. What is the maximum time step for Euler's method to remain stable in this system? Suggest, with justification, a suitable time step. [6 marks]

(*d*) Recall that a backwards stencil uses the values at the end of the timestep to determine the rate of change during that timestep. When is this usable and useful in general? Is it sensible for this application? Give modified code that implements the backwards stencil method. [6 marks]

(*e*) For the probe to drill as deep as possible, should $R(t)$ generally start small and grow larger? Justify your answer. [3 marks]