10 Algorithms (DJW)

(a) Consider a directed acyclic graph with $V$ vertices and $E$ edges.

(i) What is meant by a total order on the vertices consistent with the edges? [2 marks]

(ii) Describe an $O(E + V)$ algorithm to compute such a total order. [3 marks]

(b) Consider a directed graph with non-negative edge costs and with a given start vertex $s$.

(i) Dijkstra’s algorithm computes distances from $s$ to every other vertex. Give pseudocode for Dijkstra’s algorithm. [4 marks]

(ii) Dijkstra’s algorithm can be implemented using a Fibonacci heap. State the complexity of using this implementation. Justify your answer carefully. [Note: Your answer should include mention of amortized costs.] [4 marks]

(c) Consider a directed acyclic graph with non-negative edge costs and with a given start vertex $s$.

(i) Devise an algorithm to compute distances from $s$ in $O(E + V)$ time. Justify why your algorithm is correct. [4 marks]

(ii) Explain, with an example, why Dijkstra’s algorithm might take $\Omega(V \log V)$ time. [3 marks]