1 Foundations of Computer Science (LCP)

A one-person game (such as Rubik’s cube, or peg solitaire) has a finite number of possible states, some of which count as winning. A move is a step from one state to another. From each given state, the player can choose from a set of (zero or more) possible next moves. We call a state winnable if a winning state can be reached from it in zero or more moves.

For simplicity, assume that states are coded as integers. Also assume that we are given functions \(\text{winning}(x)\) returning true or false and \(\text{next}(x)\) returning the list of states that can be reached in one move from state \(x\).

(a) The following code is an attempt to implement the notion of winnable:

\[
\begin{align*}
\text{fun} & \ \text{exists} \ p \ [\] \ = \ \text{false} \\
& \mid \text{exists} \ p \ (x::xs) = p \ x \ \text{orelse} \ \text{exists} \ p \ xs; \\
\text{fun} \ \text{winnable} \ x = \ \text{winning} \ x \ \text{orelse} \ \text{exists} \ \text{winnable} \ (\text{next} \ x);
\end{align*}
\]

Briefly explain how this code works. Also describe its main limitation: how it can fail to find a winning state that is only a few moves away. Illustrate this point by giving specific definitions of \(\text{winning}\) and \(\text{next}\). [5 marks]

(b) Modify the code above to yield the function \(\text{winpath} \ x\), which returns the list of states from \(x\) to the winning state found or, alternatively, the empty list to indicate that no winning state was found. [4 marks]

(c) Sometimes we are only interested in a winnable state if it is only a few moves away from the current state. Modify your solution from part (b) to obtain the function \(\text{bounded\\_winpath} \ n \ x\), which looks for winning states that are at most \(n\) moves away from \(x\). [3 marks]

(d) Use your solution from part (c) to obtain the function \(\text{new\\_winpath} \ x\), which has the same objective as \(\text{winpath} \ x\), but without the limitation mentioned in part (a). Briefly explain why the limitation no longer applies and the price that has been paid for this. [5 marks]

(e) Briefly outline an alternative approach to correcting the limitation mentioned in part (a), using the notion of a queue. What are the advantages and drawbacks of this approach? [3 marks]

For full credit, code should be concise and clear. Exceptions may be useful in this but are not required.