7 Information Theory (JGD)

(a) A continuous communication channel adds Gaussian white noise to signals transmitted through it. The ratio of signal power to noise power is 30 decibels, and the frequency bandwidth of this channel is 10 MHz. Roughly what is the information capacity $C$ of this channel, in bits/second? [5 marks]

(b) Explain the comb function, $\text{comb}(t) = \delta_X(t)$, its role in the sampling theorem, its self-Fourier property, and the constraint on the spacing of the comb’s tines that is required in both the signal domain and consequently in the Fourier domain in order to reconstruct exactly, from discrete samples, a signal having no frequency components higher than $W$. [10 marks]

(c) Explain what Logan’s Theorem asserts about the richness of the zero-crossings in signals strictly bandlimited to one octave (as illustrated in the figure below). Consider an amplitude-modulated signal such as $f(t) = [1 + a(t)] c(t)$, where $c(t)$ is a pure sinusoidal carrier wave and its modulating function is $[1 + a(t)] > 0$. What would Logan’s Theorem say about the information contained in its zero-crossings? Name one intended application, and at least one algorithmic difficulty, of Logan’s theorem.

[5 marks]