3 Artificial Intelligence II (SBH)

Let $X$, $Y$ and $Z$ be random variables. Denote by $X \perp Y \mid Z$ that $X$ and $Y$ are conditionally independent given $Z$.

(a) Give two definitions of what it means to say that $X \perp Y \mid Z$, and prove that they are equivalent. [4 marks]

(b) Prove that if $X_1, X_2 \perp Y_1, Y_2 \mid Z$ then $X_1, X_2 \perp Y_1 \mid Z$. [3 marks]

(c) For each of the following Bayesian networks, state whether $X \perp Y \mid Z$ and justify your answer. [7 marks]

(d) Give a detailed explanation of how a Markov chain Monte Carlo algorithm might be used to estimate an arbitrary inference for the following Bayesian network. [6 marks]