2 Advanced Graphics (PB)

A force function $F : \mathbb{R}^3 \to \mathbb{R}$ takes a 3D point and returns a scalar representing a value of force. Force functions are the fundamental building blocks of metaball modelling.

We will build an implicit surface renderer which takes as input a set of force functions $\{F_1(P), \ldots, F_n(P)\}$ and renders the set of all points $P$ in space where the forces of the functions sum to a threshold: the 3D isosurface such that $\sum F_i(P) = 0.5$.

(a) Using pseudocode, give a force function $Sphere(P)$ which will render a unit sphere centred on $(0, 0, 0)$. [Figure 1] [2 marks]

(b) Using pseudocode, give a force function $Cube(P)$ which will render an axis-aligned cube of edge length 2 centred on $(1, 1, -1)$. [Figure 2] [4 marks]

(c) You now pass both $Sphere(P)$ and $Cube(P)$ to your implicit surface renderer. Depending on your choice of force functions, the seam between the cube and the sphere may be a sharp edge (to within the tolerance of your polygonalization) or a smooth blend which merges gradually from one form into the other. Which will it be, and (briefly) why? [Figures 3 and 4] [2 marks]

(d) Provide alternate formulations of $Sphere(P)$ and/or $Cube(P)$ such that if you answered ‘smooth’ to Part (c) then your answer would now be ‘sharp’, or vice-versa. [4 marks]

A spatial distortion function $S : \mathbb{R}^3 \to \mathbb{R}^3$ transforms one 3D point to another. If the points passed into the force function are modified by a spatial distortion function—that is, if we render $F(S(P))$—then the rendered isosurface will have a different shape.

For example, if we define $S(P)$ as

```python
def Point S(P):
    return new Point(P.x * 2, P.y / 2, P.z * 2);
```

then rendering the implicit surface of $Sphere(S(P))$ will yield a tall, narrow ellipsoid along the $Y$ axis. [Figure 5]

(e) Give a spatial distortion function $S(P)$ such that rendering the isosurface of $Cube(S(P))$ would render the cube centred at the origin and rotated 45 degrees around the $X$ axis. [Figure 6]

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\text{Hint: a standard rotation matrix is } \begin{pmatrix} \cos(t) & -\sin(t) \\ \sin(t) & \cos(t) \end{pmatrix}. \]

[3 marks]
(f) Define $S(P)$ as

```javascript
function Point S(P) {
    return new Point(
        P.x / 4,
        P.y * 2 / sin(P.x * PI),
        P.z * 2);
}
```

Describe and draw a sketch of the isosurface defined by $Sphere(S(P))$. [5 marks]

**Figures:**

- Figure 1: A sphere centred at $(0,0,0)$
- Figure 2: A cube of edge length 2 centred at $(1,1,-1)$
- Figure 3: A sharp join between sphere and cube
- Figure 4: A smooth blending between sphere and cube
- Figure 5: A vertical ellipsoid
- Figure 6: A tilted cube centred at $(0,0,0)$