2 Advanced Graphics (PB)

A *force function* $F : \mathbb{R}^3 \rightarrow \mathbb{R}$ takes a 3D point and returns a scalar representing a value of force. Force functions are the fundamental building blocks of metaball modelling.

We will build an implicit surface renderer which takes as input a set of force functions $\{F_1(P), \ldots, F_n(P)\}$ and renders the set of all points P in space where the forces of the functions sum to a threshold: the 3D isosurface such that $\sum F_i(P) = 0.5$.

(*a*) Using pseudocode, give a force function *Sphere(P)* which will render a unit sphere centred on $(0, 0, 0)$. [Figure 1] [2 marks]

(*b*) Using pseudocode, give a force function *Cube(P)* which will render an axis-aligned cube of edge length 2 centred on $(1, 1, -1)$. [Figure 2] [4 marks]

(*c*) You now pass both *Sphere(P)* and *Cube(P)* to your implicit surface renderer. Depending on your choice of force functions, the seam between the cube and the sphere may be a sharp edge (to within the tolerance of your polygonalization) or a smooth blend which merges gradually from one form into the other. Which will it be, and (briefly) why? [Figures 3 and 4] [2 marks]

(*d*) Provide alternate formulations of *Sphere(P)* and/or *Cube(P)* such that if you answered 'smooth' to Part (*c*) then your answer would now be 'sharp', or vice-versa. [4 marks]

A *spatial distortion function* $S : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ transforms one 3D point to another. If the points passed into the force function are modified by a spatial distortion function—that is, if we render $F(S(P))$—then the rendered isosurface will have a different shape.

For example, if we define $S(P)$ as

```
function Point S(P) {
  return new Point(P.x * 2, P.y / 2, P.z * 2);
}
```

then rendering the implicit surface of *Sphere(S(P))* will yield a tall, narrow ellipsoid along the Y axis. [Figure 5]

(*e*) Give a spatial distortion function $S(P)$ such that rendering the isosurface of *Cube(S(P))* would render the cube centred at the origin and rotated 45 degrees around the X axis. [Figure 6]

Hint: a standard rotation matrix is $\begin{pmatrix} \cos(t) & -\sin(t) \\ \sin(t) & \cos(t) \end{pmatrix}$. [3 marks]

(*f*) Define $S(P)$ as

```
function Point S(P) {
  return new Point(
      P.x / 4,
      P.y * 2 / sin(P.x * PI),
      P.z * 2);
}
```

Describe and draw a sketch of the isosurface defined by *Sphere(S(P))*.

[5 marks]

Figures:

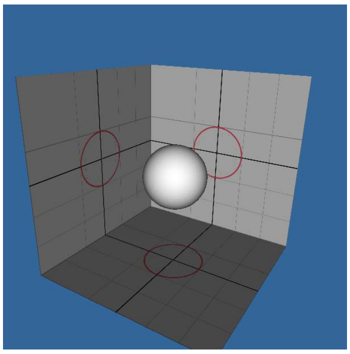

Figure 1: A sphere centred at $(0, 0, 0)$

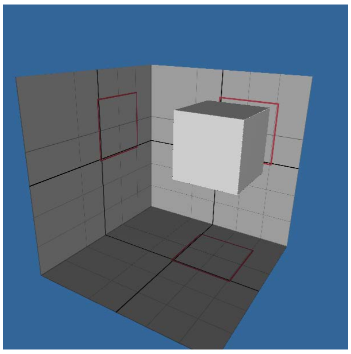

Figure 2: A cube of edge length 2 centred at $(1, 1, -1)$

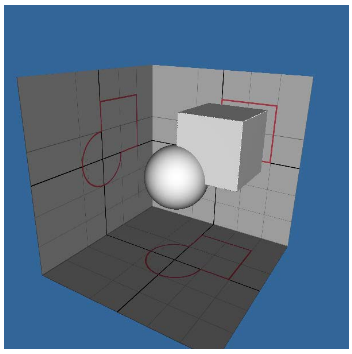

Figure 3: A sharp join between sphere and cube

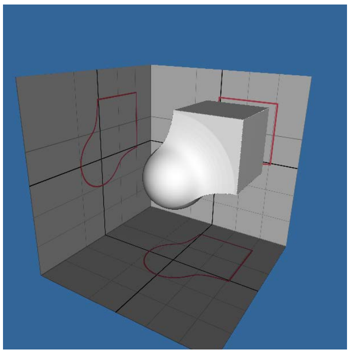

Figure 4: A smooth blending between sphere and cube

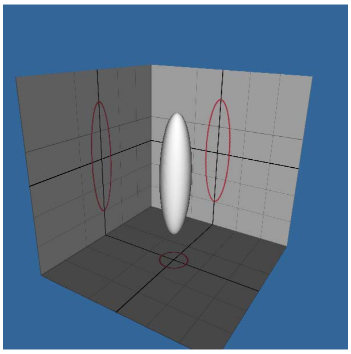

Figure 5: A vertical ellipsoid

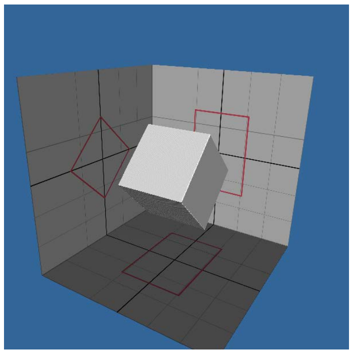

Figure 6: A tilted cube centred at $(0, 0, 0)$