5 Logic and Proof (LCP)

(a) Write brief notes on the use of clause methods to prove theorems. Include a description of an algorithm that can find a model of a set of clauses, if one exists. Illustrate your answer using the following example:

\{P, Q, \neg R\} \quad \{\neg P, R\} \quad \{\neg Q\} \quad \{P, R\}

[6 marks]

(b) For each of the following sets of clauses, either exhibit a model or show that none exists. Below, \(a\) and \(b\) are constants, while \(x, y\) and \(z\) are variables.

(i)

\{\neg P(x), Q(x, x)\}
\{\neg Q(x, y), \neg Q(y, x), R(x, y)\}
\{\neg R(x, y), \neg R(y, x)\}
\{P(a), P(b)\}

[7 marks]

(ii)

\{P(x), Q(x)\}
\{\neg P(x), Q(f(x))\}
\{P(x), \neg Q(f(x))\}
\{\neg P(x), \neg Q(x)\}

[7 marks]