4 Computation Theory (AMP)

(a) Define the terms M of the λ-*calculus* and the relation $M =_\beta M'$ of β-*conversion* between them. [6 marks]

(b) For $n \in \mathbb{N}$, what is the nth *Church numeral*? [2 marks]

(c) Consider encoding a non-empty list of λ-terms $M_1, M_2, \ldots, M_n$ as the λ-term

$$[M_1, M_2, \ldots, M_n] \triangleq \lambda x\, f.\, f\, M_1(f\, M_2 \ldots (f\, M_n\, x) \ldots)$$

where the variables x and f do not occur free in $M_1, M_2, \ldots, M_n$. Give, with justification, λ-terms Iter, Cons, Append and Nil satisfying

(i) $\mathsf{Iter}\, M\, F\, [M_1, M_2, \ldots, M_n] =_\beta F\, M_1(F\, M_2 \ldots (F\, M_n\, M))$ [2 marks]

(ii) $\mathsf{Cons}\, M\, [M_1, M_2, \ldots, M_n] =_\beta [M, M_1, M_2, \ldots, M_n]$ [3 marks]

(iii) $\mathsf{Append}\, [M_1, \ldots, M_m]\, [N_1, \ldots, N_n] =_\beta [M_1, \ldots, M_m, N_1, \ldots, N_n]$ [3 marks]

(iv) $\mathsf{Cons}\, M\, \mathsf{Nil} =_\beta [M]$, $\mathsf{Iter}\, M\, F\, \mathsf{Nil} =_\beta M$ and $\mathsf{Append}\, \mathsf{Nil}\, N =_\beta N$ [4 marks]