1 Complexity Theory (AD)

(*a*) Let $f : \mathbb{N} \to \mathbb{N}$ be a function and let $\mathrm{rng}(f)$ be defined to be the set

$$\mathrm{rng}(f) = \{y \mid f(x) = y \text{ for some } x \in \mathbb{N}\}.$$

(*i*) Define what it means to say that f is computable in polynomial time. Pay particular attention to the question of how numbers are represented as strings of symbols. [3 marks]

(*ii*) Show that if f is computable in polynomial time and increasing (i.e., for all $x \in \mathbb{N}$, $x < f(x)$), then $\mathrm{rng}(f)$ is in NP. [5 marks]

(*iii*) Show that if f is computable in polynomial time, increasing and injective, then $\mathrm{rng}(f)$ is in UP. [5 marks]

(*b*) Let $A \subseteq \mathbb{N}$ be defined as the following set of numbers

$$A = \{x \mid x = pq \text{ for distinct prime numbers } p \text{ and } q\}.$$

Prove that A is in NP and in co-NP. [7 marks]