1 Complexity Theory (AD)

(a) Let \( f : \mathbb{N} \rightarrow \mathbb{N} \) be a function and let \( \text{rng}(f) \) be defined to be the set
\[
\text{rng}(f) = \{ y \mid f(x) = y \text{ for some } x \in \mathbb{N} \}.
\]

(i) Define what it means to say that \( f \) is computable in polynomial time. Pay particular attention to the question of how numbers are represented as strings of symbols. [3 marks]

(ii) Show that if \( f \) is computable in polynomial time and increasing (i.e., for all \( x \in \mathbb{N} \), \( x < f(x) \)), then \( \text{rng}(f) \) is in NP. [5 marks]

(iii) Show that if \( f \) is computable in polynomial time, increasing and injective, then \( \text{rng}(f) \) is in UP. [5 marks]

(b) Let \( A \subseteq \mathbb{N} \) be defined as the following set of numbers
\[
A = \{ x \mid x = pq \text{ for distinct prime numbers } p \text{ and } q \}.
\]
Prove that \( A \) is in NP and in co-NP. [7 marks]