

10 Discrete Mathematics (IML)

- (a) Let $\Sigma = \{a, b, c\}$. Consider each of the subsets of Σ^* defined by the following groups of axioms and rules, and for each prove or disprove that $\#_a(u) \geq \#_b(u)$ for all $u \in \Sigma^*$, where $\#_x(u)$ is the number of occurrences of the symbol x in the string u .

$$(i) \quad \frac{}{\epsilon} \quad , \quad \frac{u}{aaub} \quad , \quad \frac{u,v}{ucv} \quad \text{for all } u, v \in \Sigma^*$$

$$(ii) \quad \frac{}{a} \quad , \quad \frac{u}{au} \quad , \quad \frac{u}{ubc} \quad \text{for all } u \in \Sigma^*$$

[6 marks]

- (b) For each of the subsets in part (a), indicate with justification whether they are regular languages.

Note: Complete proofs are not necessary but you should clearly outline any proof strategy. [10 marks]

- (c) For two regular expressions r and s and an alphabet Σ , define $r \& s$ to match a string in Σ^* if both r and s do. Given Kleene's Theorem, sketch a proof that the set of strings matched by $r \& s$ is a regular language for any regular expressions r and s . [4 marks]