6 Numerical Methods (DJG)

A picnicker brings hot black coffee and cold milk in two identical insulated flasks and then mixes them for his drink. His friend claims that the drink would have ended up the same temperature if he had mixed the two at home and brought one flask.

Note: The temperature of an object is the heat energy within it divided by its heat capacity. The rate of heat energy flow from a hotter to a cooler object is their temperature difference divided by their insulation resistance. When two fluids are mixed the resultant temperature is the sum of their initial temperatures weighted by their proportions.

(*a*) Give a suitable state vector for a simple, finite-difference, time domain simulation of the drink system. [3 marks]

(*b*) List the initial values and any other parameters that are needed for the simulation. Sketch pseudocode for each of the two scenarios. Assume constant ambient temperature and state any further assumptions. [7 marks]

(*c*) How would you select a fixed time step for these two simulations or should the time steps be adaptive? What accuracy might you expect to achieve? Is the choice of time step likely to affect whether the friend is proved right or wrong? [3 marks]

(*d*) Suppose the two-flask simulation were phrased in flow-matrix form. What determines how many rows the matrix would have? Is this a sensible approach to modelling the system? [4 marks]

(*e*) Why is backwards stability normally a useful property of numerical methods? Does that notion apply to this simulation? [3 marks]