## 2 Computer Systems Modelling (RJG)

Consider a birth death model with birth rates $\lambda_n$ in states $n = 0, 1, \ldots$ and death rates $\mu_n$ in states $n = 1, 2, \ldots$.

(*a*) State the detailed balance conditions for the equilibrium probability $p_n$ of being in state $n$, for $n = 0, 1, \ldots$ and explain how they are derived. [2 marks]

(*b*) Using the detailed balance conditions derive an expression $p_n$ and state any condition needed to ensure that the equilibrium distribution exists. [4 marks]

(*c*) Now consider the M/M/1 queue with arrival rate $\lambda$ and service rate $\mu$ and explain how it can be modelled as a birth death model. [2 marks]

(*d*) For the M/M/1 queue derive the form of $p_n$, the equilibrium distribution of the number of customers present, and state any conditions needed to ensure the existence of the equilibrium distribution. Derive the mean value of the equilibrium distribution. [4 marks]

(*e*) Now consider a M/M/1 model with the modification that customers waiting in the queue are impatient and will only wait for an exponentially distributed amount of time with rate parameter $\theta$ before departing the queue without service.

(*i*) Explain how you could modify your birth death model in this situation and write an expression for $p_n$. [4 marks]

(*ii*) Let $\alpha$ be the probability that a customer receives service and derive an expression for $\alpha$ using Little's law applied to the server. You may leave your expression in terms of the value $p_0$. [4 marks]