8 Quantum Computing (AD)

(*a*) Consider the following two-qubit quantum state, $|\phi\rangle$.

$$\frac{\sqrt{2}}{3\sqrt{3}}|00\rangle - \frac{1}{\sqrt{6}}|01\rangle + \frac{2i\sqrt{2}}{3\sqrt{3}}|10\rangle - \frac{5i}{3\sqrt{6}}|11\rangle$$

(*i*) What are the probabilities of outcomes 0 and 1 if the first qubit of $|\phi\rangle$ is measured?

(*ii*) What are the probabilities of outcomes 0 and 1 if the second qubit of $|\phi\rangle$ is measured?

(*iii*) What is the state of the system after the first qubit of $|\phi\rangle$ is measured to be a 0?

(*iv*) What is the state of the system if the second qubit of $|\phi\rangle$ is measured to be a 1?

(*v*) What are the probabilities of outcomes 0 and 1 if the second qubit of the system is measured, after the first qubit of $|\phi\rangle$ has been measured to be 0?

(*vi*) What are the probabilities of outcomes 0 and 1 if the first qubit of the system is measured, after the second qubit of $|\phi\rangle$ has been measured to be 1?

[2 marks each]

(*b*) The two qubit *quantum Fourier transform* is given by the following matrix.

$$F_2 = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix}$$

Sketch a circuit for implementing the operator F_2 using any combination of 1-qubit Hadamard gates; 1-qubit Pauli gates; 2-qubit C-NOT gates; controlled phase shifts and swap gates (the *swap gate* S is defined by $S|xy\rangle = |yx\rangle$). Briefly explain your circuit. [8 marks]