

1 Complexity Theory (AD)

(a) Give precise definitions of each of the following:

- (i) the complexity class NP;
- (ii) polynomial-time reduction; and
- (iii) NP-complete problem.

[3 x 2 marks]

(b) An instance of a *linear programming* problem consists of a set  $X = \{x_1, \dots, x_n\}$  of variables and a set of *integer constraints*, each of which is of the form

$$\sum_{1 \leq i \leq n} a_i x_i \leq b,$$

where each  $a_i$  and  $b$  is an integer.

The 0-1 Integer Linear Programming feasibility problem (ILP) is, to determine, given such a linear programming problem, whether there is an assignment of values from the set  $\{0, 1\}$  to the variables in  $X$  so that substituting these values in the constraints leads to all constraints being simultaneously satisfied.

- (i) Consider a *clause*  $c$ , i.e. a disjunction of Boolean literals. Show how such a clause can be converted to an integer constraint which has a  $\{0, 1\}$ -solution if, and only if,  $c$  is satisfiable. [4 marks]
- (ii) Use part (b)(i) to show that there is a polynomial-time reduction from the problem CNF-SAT to ILP. [4 marks]
- (iii) Is there a polynomial-time reduction from ILP to CNF-SAT? Justify your answer. [4 marks]
- (iv) What can you conclude about the complexity of ILP? [2 marks]