This question explores how we might use Prolog to match Regular Expressions. We represent the sequence to be matched in Prolog using a list of atoms. For example, \texttt{aaba} would be represented as the list \texttt{[a,a,b,a,end]} using the atom \texttt{end} to encode the end of the string explicitly.

A simple scheme for writing Regular Expressions uses a single character as itself and uses the plus symbol (+) to indicate that there should be one or more instances of the previous character. In this question we consider the Regular Expression \texttt{a\^{+}b\^{+}a} which means one or more occurrences of \texttt{a}, followed by one or more occurrences of \texttt{b}, followed by a single occurrence of \texttt{a}.

(a) Draw a state machine which is capable of matching the Regular Expression \texttt{a\^{+}b\^{+}a}. Clearly indicate the start and finish states. [2 marks]

(b) Define a predicate \texttt{t(A,B,C)} which encodes the transitions of your state machine. \texttt{t(A,B,C)} should be true if there is a transition from state \texttt{A} to state \texttt{B} when we see a character \texttt{C}. Indicate which of your definitions are facts and which are rules. [2 marks]

(c) Predicates for testing a solution do not always work when generating solutions. Demonstrate this by writing a Prolog predicate \texttt{matches(L)} which tests if \texttt{L} represents a string which matches the Regular Expression \texttt{a\^{+}b\^{+}a}. [5 marks]

(d) Why is your predicate \texttt{matches(L)} not a good solution for generating strings matching the Regular Expression \texttt{a\^{+}b\^{+}a}? Describe a specific execution path in which a problem can occur. [3 marks]

(e) Describe a better strategy for generating strings matching the Regular Expression \texttt{a\^{+}b\^{+}a} and provide an implementation. Clearly explain the approach you are using and why it is a sensible choice. [8 marks]