9 Algorithms (TMS)

(*a*) Consider the two standard representations of directed graphs: the adjacency-list representation and the adjacency-matrix representation. Find a problem that can be solved more efficiently in the adjacency-list representation than in the adjacency-matrix representation, and another problem that can be solved more efficiently in the adjacency-matrix representation than in the adjacency-list representation. [4 marks]

(*b*) Prove or disprove (by giving a counter-example) the following claim: If a directed graph G contains a path from a vertex u to a vertex v, then any depth-first search must result in $v.d \leq u.f$, where $.d$ is the discovery time and $.f$ the finishing time. [4 marks]

(*c*) We are given an undirected, connected graph $G = (V, E)$ with edge-weights $w : E \to \mathbb{R}^+$ and a minimum spanning tree T of G. How would you update your minimum spanning tree T in each of the following three cases? Specify the runtime of your algorithm and give a proof that the returned tree is indeed a minimum spanning tree.

(*i*) We increase the weight of an edge e which is not in T. [3 marks]

(*ii*) We decrease the weight of an edge e which is in T. [3 marks]

(*iii*) We add a new edge e with weight $w(e)$ to G. The weight $w(e)$ is arbitrary, but for simplicity you may assume that after adding the edge e no two edges in G have the same weight. [6 marks]