9 Algorithms (TMS)

(a) Consider the two standard representations of directed graphs: the adjacency-list representation and the adjacency-matrix representation. Find a problem that can be solved more efficiently in the adjacency-list representation than in the adjacency-matrix representation, and another problem that can be solved more efficiently in the adjacency-matrix representation than in the adjacency-list representation. [4 marks]

(b) Prove or disprove (by giving a counter-example) the following claim: If a directed graph $G$ contains a path from a vertex $u$ to a vertex $v$, then any depth-first search must result in $v.d \leq u.f$, where $d$ is the discovery time and $f$ the finishing time. [4 marks]

(c) We are given an undirected, connected graph $G = (V, E)$ with edge-weights $w : E \rightarrow \mathbb{R}^+$ and a minimum spanning tree $T$ of $G$. How would you update your minimum spanning tree $T$ in each of the following three cases? Specify the runtime of your algorithm and give a proof that the returned tree is indeed a minimum spanning tree.

(i) We increase the weight of an edge $e$ which is not in $T$. [3 marks]

(ii) We decrease the weight of an edge $e$ which is in $T$. [3 marks]

(iii) We add a new edge $e$ with weight $w(e)$ to $G$. The weight $w(e)$ is arbitrary, but for simplicity you may assume that after adding the edge $e$ no two edges in $G$ have the same weight. [6 marks]