8 Algorithms (FMS)

(a) Explain the greedy strategy in algorithm design. To what problems does it apply? [3 marks]

(b) If a problem can be solved with both dynamic programming and a greedy algorithm, what are the advantages of using one or the other? [2 marks]

(c) An imaginary post office machine must issue decorative stamps adding up to a given amount of \( p \) pence. Its goal is to minimize the number of postage stamps issued, and the machine always has as many stamps as needed.

(i) Let the set of available denominations for the stamps be \( D = \{1p, 5p, 25p, 50p, £1, £2\} \). Can this problem be solved using bottom-up dynamic programming? If so, clearly describe your algorithm and determine its complexity. If not, prove that it cannot be done. [5 marks]

(ii) Let \( c_1 < c_2 < \cdots < c_n \) be \( n \) stamp denominations. Prove that if each \( c_i \) (a positive integer) is a multiple of \( c_{i-1} \) for every \( i = 2, \ldots, n \) then the greedy strategy applied to the set \( D = \{c_1, c_2, \ldots, c_n\} \) finds the optimal solution for any amount \( p \) that is a multiple of \( c_1 \). [7 marks]

(iii) Provide a set of denominations for stamps \( D \) and an amount of pence \( p \) for which the greedy strategy fails to give an optimal solution, \( p \) being a multiple of the smallest denomination in \( D \). Show what solution the greedy strategy would find and what the optimal solution is. [3 marks]