6 Information Theory and Coding (JGD)

(a) A two state Markov process emits the letters \{A, B, C, D, E\} with the probabilities shown for each state. Changes of state can occur when some of the symbols are generated, as indicated by the arrows.

\[ \begin{array}{c|c|c}
\text{State 1} & 1/2 & 1/4 \\
\text{State 2} & 1/4 & 1/8 \\
\end{array} \]

(i) What are the state occupancy probabilities? \[1\text{ mark}\]

(ii) What is the probability of the letter string AD being emitted? \[1\text{ mark}\]

(iii) What is the entropy of State 1, what is the entropy of State 2, and what is the overall entropy of this symbol generating process? \[5\text{ marks}\]

(b) A fair coin is secretly flipped until the first head occurs. Let \(X\) denote the number of flips required. The flipper will truthfully answer any “yes-no” questions about his experiment, and we wish to discover thereby the value of \(X\) as efficiently as possible.

(i) What is the most efficient possible sequence of such questions? Justify your answer. \[2\text{ marks}\]

(ii) On average, how many questions should we need to ask? Justify your answer. \[2\text{ marks}\]

(iii) Relate the sequence of questions to the bits in a uniquely decodable prefix code for \(X\). \[1\text{ mark}\]

(c) Define complex Gabor wavelets, restricting yourself to one-dimensional functions if you wish, and list four key properties that make such wavelets useful for encoding and compressing information, as well as for pattern recognition. Explain how their self-Fourier property and their closure under multiplication (i.e. the product of any two of them is yet again a Gabor wavelet) gives them also closure under convolution. Mention one disadvantage of such wavelets for reconstructing data from their projection coefficients. \[8\text{ marks}\]