2 Computer Systems Modelling (RJG)

(*a*) Consider a Poisson process with rate $\lambda > 0$. Let X_1 be the time of the first event and let X_i be the time between events $(i-1)$-st and i for $i = 2, 3, \ldots$

(*i*) Derive the joint probability distribution of (X_1, X_2). [5 marks]

(*ii*) Let $S_n = \sum_{i=1}^{n} X_i$. Derive the probability density function of S_n and give expressions for the mean and variance of S_n. [5 marks]

(*b*) (*i*) Describe what is meant by a FCFS $M/G/1$ queueing system. Your description should include a clear statement of the probabilistic assumptions. [5 marks]

(*ii*) Suppose that you are given a log of timestamps for the arrival and departure events observed in an alleged simulation of a FCFS $M/G/1$ queue with a given general service time distribution. Describe the statistical tests that you would perform on the logged data to test whether the modelling assumptions are satisfied. [5 marks]