13 Types (AMP)

(a) Give an account of the Polymorphic Lambda Calculus (PLC). You should define the PLC types, expressions and typing environments, give the PLC typing rules, and define the relations of beta-reduction and beta-conversion. Explain why beta-conversion is a decidable relation for expressions that are typeable.

[10 marks]

(b) Find, with justification, a PLC type $\tau$ for which the following typings are both provable:

$$\{} \vdash \Lambda \alpha, \beta (\lambda x : \alpha (\lambda y : \beta (\Lambda \gamma (\lambda z : \tau (z x y)))))) : \forall \alpha, \beta (\alpha \to (\beta \to \forall \gamma (\tau \to \gamma)))$$

$$\{} \vdash \Lambda \alpha, \beta (\lambda z : \forall \gamma (\tau \to \gamma)(\alpha x : \alpha (\beta (x)))))) : \forall \alpha, \beta ((\forall \gamma (\tau \to \gamma)) \to \alpha)$$

[5 marks]

(c) Give infinitely many different closed PLC expressions in beta-normal form of type $\forall \alpha ((\alpha \to \alpha) \to (\alpha \to \alpha))$.

[2 marks]

(d) Use your answer to part (a) to show that there is no closed PLC expression $Y$ of type $\forall \alpha ((\alpha \to \alpha) \to \alpha)$ for which the beta-reduction $Y \alpha f \to f(Y \alpha f)$ holds.

[3 marks]