8 Temporal Logic and Model Checking (MJCG)

From Wikipedia: “Tic-tac-toe (or Noughts and Crosses, Xs and Os) is a paper-and-pencil game for two players, X and O, who take turns marking the spaces in a 3×3 grid. The player who succeeds in placing three respective marks in a horizontal, vertical, or diagonal row wins the game.” For example, X is the first player in both example games shown below; the first game is won by the X, the second is drawn.

(a) This part of the question asks you to define a Kripke structure $M = (S, S_0, R, L)$ to model Tic-tac-toe. Assume the set $AP$ consists of atomic propositions $\text{Start}(p)$ and $\text{Has}(i, v)$. $\text{Start}(p)$ means player $p$ starts, where $p \in \{0, 1\}$ represents a player: 0, 1 represent 0, X, respectively. $\text{Has}(i, v)$ means space $i$ contains value $v$, where $i \in \{1, \ldots, 9\}$ names a grid space and $v \in \{0, 1, 2\}$ represents the state of a space: 0, 1, 2 represent 0, X, empty-space, respectively.

(i) Specify a suitable representation $S$ of states. [2 marks]

(ii) Specify the set of initial states $S_0$. [2 marks]

(iii) Specify a transition relation $R$ to model the moves in the game. [6 marks]

(iv) Specify a labelling function $L$ to define which atomic propositions hold in each state. [2 marks]

(b) In a suitable temporal logic, which you should name, devise and explain a formula $\psi$ such that $M \models \psi$ if and only if the first player can always win or draw, no matter how the second player plays. [8 marks]