1 Advanced Graphics (NAD)

(*a*) From a uniform knot vector, derive the basis functions of a uniform quadratic B-spline. That is, derive $N_{1,3}$ from the knot vector and also show how $N_{i,3}$ is related to $N_{1,3}$ for arbitrary i. [7 marks]

(*b*) It is known that the uniform quadratic B-spline curve is continuous in its first derivative but that it is not guaranteed continuous in its second derivative. Prove that $N_{1,3}$ is discontinuous in its second derivative at one or more points. [4 marks]

(*c*) The Chaikin corner-cutting subdivision scheme is related to the uniform quadratic B-spline in that the limit curve of the Chaikin scheme is the uniform quadratic B-spline curve generated from the original control points, $(P^0_0, P^0_1, P^0_2, \ldots)$.

Given a set of control points, $(P^n_0, P^n_1, P^n_2, \ldots)$, at subdivision level n, the Chaikin scheme generates a new set of control points at level $n+1$ by two rules:

$$
\begin{aligned}
P^{n+1}_{2i} &= \tfrac{3}{4}P^n_i + \tfrac{1}{4}P^n_{i+1} \\
P^{n+1}_{2i+1} &= \tfrac{1}{4}P^n_i + \tfrac{3}{4}P^n_{i+1}
\end{aligned}
$$

Consider the sequence $P^0_i, P^1_{2i}, P^2_{4i}, \ldots, P^n_{2^n i}, \ldots$. From your answer to part (*a*), or otherwise, determine $\lim_{n\to\infty} P^n_{2^n i}$ in terms of the original control points. [4 marks]

(*d*) The univariate Chaikin curve scheme described in part (*c*) can be generalised to a bivariate scheme that generates surfaces. Explain how it can be generalised to generate surfaces from an arbitrary mesh of control points, paying attention to both the regular and the extraordinary cases. [5 marks]