

9 Information Theory and Coding (JGD)

- (a) Suppose that women who live beyond the age of 80 outnumber men in the same age group by three to one. How much information, in bits, is gained by learning that a person who lives beyond 80 is male? [2 marks]
- (b) Consider  $n$  discrete random variables, named  $X_1, X_2, \dots, X_n$ , of which  $X_i$  has entropy  $H(X_i)$ , the largest being  $H(X_L)$ . What is the upper bound on the joint entropy  $H(X_1, X_2, \dots, X_n)$  of all these random variables, and under what condition will this upper bound be reached? What is the lower bound on the joint entropy  $H(X_1, X_2, \dots, X_n)$ ? [3 marks]
- (c) If discrete symbols from an alphabet  $S$  having entropy  $H(S)$  are encoded into blocks of length  $n$  symbols, we derive a new alphabet of symbol blocks  $S^n$ . If the occurrence of symbols is independent, then what is the entropy  $H(S^n)$  of this new alphabet of symbol blocks? [2 marks]
- (d) Consider an asymmetric communication channel whose input source is the binary alphabet  $X = \{0, 1\}$  with probabilities  $\{0.5, 0.5\}$  and whose outputs  $Y$  are also this binary alphabet  $\{0, 1\}$ , but with asymmetric error probabilities. Thus an input 0 is flipped with probability  $\alpha$ , but an input 1 is flipped with probability  $\beta$ , giving this channel matrix  $p(y_k|x_j)$ :

$$\begin{pmatrix} 1 - \alpha & \alpha \\ \beta & 1 - \beta \end{pmatrix}$$

- (i) Give the probabilities of both outputs,  $p(Y = 0)$  and  $p(Y = 1)$ . [2 marks]
- (ii) Give all the values of  $(\alpha, \beta)$  that would maximise the capacity of this channel, and state what that capacity then would be. [3 marks]
- (iii) Give all the values of  $(\alpha, \beta)$  that would minimise the capacity of this channel, and state what that capacity then would be. [3 marks]
- (e) In order for a variable length code having  $N$  codewords with bit lengths  $\{n_1, n_2, n_3, \dots, n_N\}$  to satisfy the prefix property, what condition must be satisfied? (Express the condition, but do not try to prove it.) [1 mark]
- (f) The information in continuous signals which are strictly bandlimited (lowpass or bandpass) is *quantised*, in that such continuous signals can be completely represented by a finite set of discrete samples. Describe two theorems about how discrete samples suffice for exact reconstruction of continuous bandlimited signals, even at all the points between the sampled values. [4 marks]