COMPUTER SCIENCE TRIPOS Part IB – 2014 – Paper 6

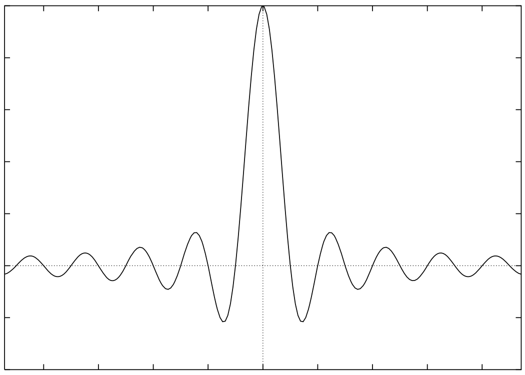
7 Mathematical Methods for Computer Science (JGD)

(*a*) Using complex exponentials, prove the following trigonometric identity, which describes the multiplicative modulation of one cosine wave by another as being simply the sum of a different pair of cosine waves:

$$\cos(ax)\cos(bx) = \frac{1}{2}\cos((a+b)x) + \frac{1}{2}\cos((a-b)x)$$

[3 marks]

(*b*) The function $\text{sinc}(x) = \dfrac{\sin(\pi x)}{\pi x}$ for $x \neq 0$ as plotted here plays an important role in the Sampling Theorem. By considering its Fourier transform, show that this function is unchanged in form after convolution with itself, and show that it even remains unchanged in form after convolution with any higher frequency sinc function $\text{sinc}(ax)$ for $a > 1$, but that if $0 < a < 1$, then the result is instead that lower frequency sinc function $\text{sinc}(ax)$.



[5 marks]

(*c*) Let V be an inner product space spanned by an orthonormal system of vectors $\{e_1, e_2, \ldots, e_n\}$ so that $\forall i \neq j$ the inner product $\langle e_i, e_j \rangle = 0$, but every e_i is a unit vector so that $\langle e_i, e_i \rangle = 1$. We wish to represent a data set consisting of vectors $u \in \text{span}\{e_1, e_2, \ldots, e_n\}$ in this space as a linear combination of the orthonormal vectors: $u = \sum_{i=1}^{n} a_i e_i$. Derive how the coefficients a_i can be determined for any vector u, and comment on the computational advantage of representing the data in an orthonormal system. [7 marks]

(*d*) Show how a generating (or "mother") wavelet $\Psi(x)$ can spawn a family of "daughter" wavelets $\Psi_{jk}(x)$ by simple shifting and scaling operations, and explain the advantages of representing continuous functions in terms of such a family of self-similar dilates and translates of a mother wavelet. [5 marks]