COMPUTER SCIENCE TRIPOS Part IB – 2014 – Paper 6

3 Computation Theory (AMP)

(*a*) Explain how to code register machine programs P as numbers $\ulcorner P \urcorner \in \mathbb{N}$ so that each $e \in \mathbb{N}$ can be decoded to a unique register machine program $\mathit{prog}(e)$. [10 marks]

(*b*) Find a number $e_1 \in \mathbb{N}$ for which $\mathit{prog}(e_1)$ is a register machine program for computing the function $\mathit{one} \in \mathbb{N} \to \mathbb{N}$ with $\mathit{one}(x) = 1$ for all $x \in \mathbb{N}$. [2 marks]

(*c*) Why is it important for the theory of computation that the functions involved in the coding and decoding given in part (*a*) are themselves register machine computable? (You are not required to prove that they are computable.) [2 marks]

(*d*) Define what it means for a set of numbers $S \subseteq \mathbb{N}$ to be register machine *decidable*. [2 marks]

(*e*) Let $\varphi_e \in \mathbb{N} \rightharpoonup \mathbb{N}$ denote the partial function of one argument computed by the register machine with program $\mathit{prog}(e)$. Prove that $\{e \in \mathbb{N} \mid \varphi_e = \mathit{one}\}$ is register machine undecidable (where one is the function mentioned in part (*b*)). State carefully any standard results that you use in your proof. [4 marks]