This question relates to binary constraint satisfaction problems (CSPs). A CSP has a set $X = \{x_1, \ldots, x_n\}$ of variables, each having a domain $D_i = \{v_1, \ldots, v_{n_i}\}$ of values. In addition, a CSP has a set $C = \{C_1, \ldots, C_m\}$ of constraints, each relating to a subset of $X$ and specifying the allowable combinations of assignments to the variables in that subset.

(a) Give a general definition of a solution to a CSP. [1 mark]

(b) Given a binary CSP, define what it means for a directed arc $x_i \rightarrow x_j$ between variables $x_i$ and $x_j$ to be arc consistent. [2 marks]

(c) Give an example of how a directed arc $x_i \rightarrow x_j$ can fail to be arc consistent. Explain how this can be fixed. [2 marks]

(d) Describe the AC-3 algorithm for enforcing arc consistency. [5 marks]

(e) Prove that the time complexity of the AC-3 algorithm is $O(n^2d^3)$ where $d$ is the size of the largest domain. [3 marks]

(f) Suggest a way in which the concept of arc consistency, also known as 2-consistency can be extended to sets of three, rather than two variables. In the remainder of the question we will refer to this as 3-consistency. [1 mark]

(g) Give an example of how a set of three variables might fail to be 3-consistent, and show how 3-consistency might then be imposed. [2 marks]

(h) Suggest a modified version of the AC-3 algorithm that can be used to enforce 3-consistency. [4 marks]