10 Discrete Mathematics (AMP)

(a) For each symbol $x$ in the alphabet $\Sigma = \{a, b, c\}$, let $O_x$ be the language over $\Sigma$ consisting of all strings that contain an odd number of occurrences of the symbol $x$; and let $E_x$ be the language of strings over $\Sigma$ containing an even number of occurrences of the symbol $x$.

(i) Give a deterministic finite automaton whose language of accepted strings is $O_a$. [2 marks]

(ii) Give a regular expression whose language of matching strings is $O_a$. [2 marks]

(iii) Give a deterministic finite automaton whose language of accepted strings is $O_a \cap E_b$. [4 marks]

(b) $M = (Q, \Sigma, \delta, s, F)$ is a deterministic finite automaton whose set of states $Q$ has $\ell$ elements. Suppose that $M$ accepts a string $w \in \Sigma^*$ whose length $|w|$ satisfies $|w| \geq \ell$.

(i) Show that $w = u_1vu_2$ for some strings $u_1, v, u_2 \in \Sigma^*$ such that $|u_1| < \ell$, $1 \leq |v| \leq \ell$ and $M$ accepts $u_1v^n u_2$ for all $n \in \mathbb{N} = \{0, 1, 2, \ldots\}$. [7 marks]

(ii) Hence show that if infinitely many strings are accepted by $M$, then it must accept some string $w'$ with $\ell \leq |w'| < 2\ell$. [5 marks]