9 Algorithms (TMS)

(a) Explain the terms amortized analysis, aggregate analysis and potential method. [6 marks]

(b) Consider an arbitrary sequence of $n$ stack operations $\text{PUSH}(), \text{POP}()$ and $\text{MULTIPOP}(x)$ in which $\text{POP}()$ or $\text{MULTIPOP}(x)$ never attempt to remove more elements than there are on the stack. Assuming that the stack begins with $s_0$ items and finishes with $s_n$ items, determine the worst-case total cost for executing the $n$ operations as a function of $n$, $s_0$ and $s_n$. You may assume $\text{PUSH}()$ and $\text{POP}()$ cost 1 each and $\text{MULTIPOP}(x)$ costs $x$. [5 marks]

(c) Suppose we want to store a number of items in an array, but we do not know in advance how many items need to be stored. The $\text{INSERT}(x)$ operation appends an item $x$ to the array. More precisely, if the size of the array is large enough, $x$ is inserted directly at the end of the array. Otherwise, a new array of larger size is created that contains all previous items with $x$ being appended at the end. The total cost of $\text{INSERT}(x)$ is 1 in the first case, and the size of the new array in the second case.

(i) Devise a strategy which, for any integer $n$, performs any sequence of $n$ $\text{INSERT}(. )$ operations at a total cost of $O(n)$. [5 marks]

(ii) For the strategy described in (c)(i), give a proof of the cost of the algorithm using the potential method. [4 marks]