2 Computer Systems Modelling (RJG)

(*a*) Define a Poisson process of rate $\lambda > 0$. [3 marks]

(*b*) Show that the number of events, $N(t)$, of a Poisson process that occur in the fixed time interval $[0, t]$ is a random variable that has a Poisson distribution with parameter λt. [3 marks]

(*c*) Show that the inter-event times of a Poisson process form a sequence of independent random variables each distributed with an exponential distribution with parameter λ. [3 marks]

(*d*) Describe how to use the inverse transform method to simulate exponential random variables with parameter λ. [3 marks]

(*e*) Show how your simulated exponential random variables can be used to simulate Poisson random variables with parameter λ. [3 marks]

(*f*) Consider positive numbers $\lambda_1, \lambda_2, \ldots, \lambda_n$ and weight factors $\alpha_1, \alpha_2, \ldots, \alpha_n$ such that $\alpha_i \geq 0$ for $i = 1, 2, \ldots, n$ and $\sum_{i=1}^{n} \alpha_i = 1$. Show that

$$f(x) = \begin{cases} \sum_{i=1}^{n} \alpha_i \lambda_i e^{-\lambda_i x} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

is a density for a random variable and describe a procedure to simulate values from this density. [5 marks]