13 Topics in Concurrency (GW)

This question is on HOPLA and PCCS, a variant of pure CCS in which any output on a channel persists. Let A be a set of channel names ranged over by a, b, c and let $\bar{A}$ be the set of complemented channel names, $\bar{A} = \{\bar{a} \mid a \in A\}$. The set of labels $L = A \cup \bar{A}$ is ranged over by l, to which we extend complementation by taking $\bar{\bar{l}} = l$. Use α to range over $L \cup \{\tau\}$, where τ is a distinct label. The terms of PCCS follow the grammar $P ::= \mathbf{nil} \mid \bar{a} \mid a.P \mid (P_1 \parallel P_2)$. The operational semantics of PCCS is:

$$\frac{}{\bar{a} \xrightarrow{\bar{a}} \bar{a}} \qquad \frac{}{a.P \xrightarrow{a} P} \qquad \frac{P_1 \xrightarrow{\alpha} P_1'}{P_1 \parallel P_2 \xrightarrow{\alpha} P_1' \parallel P_2} \qquad \frac{P_2 \xrightarrow{\alpha} P_2'}{P_1 \parallel P_2 \xrightarrow{\alpha} P_1 \parallel P_2'} \qquad \frac{P_1 \xrightarrow{l} P_1' \quad P_2 \xrightarrow{\bar{l}} P_2'}{P_1 \parallel P_2 \xrightarrow{\tau} P_1' \parallel P_2'}$$

(*a*) Draw the transition system of the PCCS term $\bar{a} \parallel a.a.\bar{b}$. [3 marks]

(*b*) This part of the question is on HOPLA. For reference, the operational semantics of HOPLA is presented at the end of the question.

(*i*) For u of sum type, let $[u > a.x \Rightarrow t]$ abbreviate $[\pi_a(u) > .x \Rightarrow t]$. Derive a rule for the transitions of $[u > a.x \Rightarrow t]$. [2 marks]

(*ii*) Show that $[a.u > a.x \Rightarrow t] \sim t[u/x]$ and $[a.u > b.x \Rightarrow t] \sim nil$ if $a \neq b$, where nil represents the empty sum and $\sim$ is the bisimilarity of HOPLA. [4 marks]

(*c*) Write down a HOPLA term realising the parallel composition of PCCS. Use this to give an encoding of PCCS into HOPLA, specifying a HOPLA term $[\![P]\!]$ for every PCCS term P. [*Hint:* The realisation of parallel composition should be the same as that of the encoding of pure CCS into HOPLA.] [5 marks]

(*d*) Use the rules of HOPLA to show how a derivation establishing $[\![P_1 \parallel P_2]\!] \xrightarrow{\alpha.} [\![P_1' \parallel P_2]\!]$ can be constructed from a derivation of $[\![P_1]\!] \xrightarrow{\alpha.} [\![P_1']\!]$.

Explain briefly how you would show that if $P \xrightarrow{\alpha} P'$ in PCCS then $[\![P]\!] \xrightarrow{\alpha.} [\![P']\!]$ in HOPLA. In what part of the proof would the derivation that you have constructed be useful?

[6 marks]

Subject to suitable typings, HOPLA has transitions $t \xrightarrow{p} t'$ between closed terms t, t' and action p given by the following rules:

$$\frac{t[rec\, x\, t/x] \xrightarrow{p} t'}{rec\, x\, t \xrightarrow{p} t'} \qquad \frac{t_j \xrightarrow{p} t'}{\sum_{i \in I} t_i \xrightarrow{p} t'}(j \in I) \qquad \frac{}{.t \xrightarrow{.} t} \qquad \frac{u \xrightarrow{.} u' \quad t[u'/x] \xrightarrow{p} t'}{[u > .x \Rightarrow t] \xrightarrow{p} t'}$$

$$\frac{t[u/x] \xrightarrow{p} t'}{\lambda x\, t \xrightarrow{u \mapsto p} t'} \qquad \frac{t \xrightarrow{u \mapsto p} t'}{t\ u \xrightarrow{p} t'} \qquad \frac{t \xrightarrow{p} t'}{a\, t \xrightarrow{a\, p} t'} \qquad \frac{t \xrightarrow{a\, p} t'}{\pi_a(t) \xrightarrow{p} t'}$$